UQOP 2019
Uncertainty Quantification & Optimization Conference
18-20 March, Paris, France

Damage tolerance reliability assessment combining adaptive kriging and support vector machine

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Keywords: reliability, subset simulation, adaptive kriging, support vector machine, damage tolerance, aerospace engineering.

ABSTRACT

In the aerospace sector, the design approach follows the damage tolerance rules considering fracture mechanics numerical models subjected to uncertainties about geometry, material properties, loads or presence of defects. The conventional approach to treat the uncertainties is to perform deterministic evaluation setting the properties in worst conditions. Even if they ensure the strength of the component, conservative hypotheses may generate an unquantifiable over-sizing. A more rigorous way to consider uncertainties is to assess the probability of failure by setting stochastic models as input. Here, a specific procedure is defined to assess the probability of failure for damage tolerance, combining several criteria.

In the reliability context, the uncertainties are modelled by random variables X_i mapped into the standard space where all variables are defined according to an uncorrelated normal distribution law such as $u_i \sim \mathcal{N}(0;1)$. The reliability is quantified by the probability of failure p_f.

The aim of the damage tolerance approach is to ensure the component safety during its lifetime N_{target}. At each step of the crack propagation, the outputs are post-processed considering the Failure Assessment Diagram (FAD) according to R6-rule [Milne *et al.*, 1988]. The FAD margin M_{FAD} is set to qualify the state of the component at each increment of the crack. If $M_{\text{FAD}} \leq 0$, the process is stopped and the component is rejected. Otherwise, it is accepted if $N_{life} = N_{target}$. Note that, the use of linear elastic fracture mechanics hypothesis leads to obtain a qualitative information when $M_{\text{FAD}} \leq 0$. In the damage tolerance reliability assessment, FAD margin is considered as the performance function $g(\mathbf{U})$. Our goal is to assess the probability of failure limiting the damage

*Thanks to Rudy Chocat who worked as a PhD student on this topic tolerance model evaluations considering a performance function which provides quantitative information for a safe component and qualitative information for a failed one.

The reliability problem may be considered as a classification one because the standard space is divided into a failure region where the performance function $g(\mathbf{u}) < 0$, a safe region $g(\mathbf{u}) > 0$, and the limit state $g(\mathbf{u}) = 0$. The kriging regression [Echard *et al.*, 2011] or Support Vector Machine (SVM) classification [Hurtado, 2013] may be applied according to the allowable dataset. On the one hand, kriging allows taking into account the trends of the model. On the other hand, the SVM classification can treat all kinds of dataset information without any condition about the continuity of the performance function.

The idea of the proposed method is to combine kriging and SVM to take advantages of both approaches. They are associated with the Subset Simulation [Au and Beck, 2001] based on a sequence of subset steps relaxing the limit state threshold. For the first subset steps, when dataset contains positive, i.e., quantitative values only, a kriging model is built with the Adaptive Kriging (AK) [Echard *et al.*, 2011] enrichment strategy to improve the accuracy. For the last subset step, when the Design Of Experiments (DOE) is composed of both qualitative ($g(\mathbf{u}) \leq 0$) and quantitative data points ($g(\mathbf{u}) > 0$), a SVM separator is coupled with an Adaptive SVM (ASVM) [Pan and Dias, 2017] to accurately describe the limit state. The transition between the kriging and SVM phases is ensured if one of the following criteria is satisfied:

- The subset threshold is less or equal to zero, inspired by Subset Simulation,
- the DOE is composed of 2 x number of random variables failed experiments, it must prevent a deterioration of the current threshold using kriging.

This method, combining kriging and SVM by Subset Simulation, is custom-built for the damage tolerance application due to the specificity of dataset, composed of both qualitative and quantitative information. The results for low probability estimations are attractive when compared to those obtained with existing reference methods. This approach has also been successfully extended to high dimensional cases in an industrial context, with CPU-demanding finite element simulations of crack propagation.

Acknowledgments : The work presented in this paper is financially supported by CNES (French National Center for Space Studies) and ANRT (French National Association for Research and Technology) by the CIFRE convention 2015/0209.

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