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Searching for an optimal and reliable design under epistemic modelling uncertainty

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ABSTRACT

Complex systems, like satellites, require a deliberate process to develop from an idea to the actual product. Commonly, such a development process is divided to multiple phases where in each phase we consider multiple design configurations. But in the early phases of this process, we usually suffer from a lack of knowledge about models of the expected system behaviour, which will be more accurately addressed only in later design phases by refining the mathematical models and collecting experimental data. Nevertheless, in order to choose the studies which are supposed to eventually predict the system behaviour with reasonable accuracy, and also to plan for other actions necessary for the final system deployment, we need to be able to predict, to some extent, which of the possible designs from the initial design space might lead to desirable solutions and, more importantly, which will not. The issue with answering these questions lies in impossibility to construct a *reliable* precise predictive model from the information available at the initial design phase. Such an attempt usually requires us to rely on several strong and unjustifiable assumptions, like that of precise stochastic models describing the system or component responses. In our approach, we, instead, base our assessments on genuinely imprecise models which allow to better model our initial lack of knowledge.
A reasonable *optimal* design would need to satisfy two basic criteria. First, we would like it to perform better that adversarial designs based on whatever performance criteria we deem desirable. Secondly, we would like the system to perform at all. When a system becomes more complex, the amount of possible failure modes (events which would render the system in-operational) generally

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increases. It is therefore crucial to also address such possibility in our models and choose the design which would conclude in a reasonably *reliable* system. Reliability of a system can also be increased based on our design choices, for example by using more robust components or backing up some of the crucial sub-systems, and, similarly as with the performance, we can construct mathematical models for its prediction.

Our main concern in this contribution therefore lies in how to formulate a mathematical problem of finding a reliable system design when the system performance is subjected to uncertainty, and how to do the same in case when our models can give us only imprecise answers which results in loosing the total ordering of the designs based on their quality.

Optimisation under uncertainty may be addressed in several ways. One is to make decisions, choosing an optimal design, based on the expected (mean) performance of the system [deFinetti]. Such an approach also allows us to reduce the problem of performance-based-design into a problem of optimising a real function in the case when uncertainties are described by precise stochastic models. It also allows us to naturally include the reliability because the state of a system, functional or failed, directly influences its performance and therefore its expected performance. But the expected value approach is unable to properly capture the variation of performance indices. In case when we would be constructing a large amount of identical *in-mean-optimal* systems, they would perform optimally as a group, but in the case of a *one-of-a-kind* system, like a satellite, we would like to 'ensure' that it would perform as well as possible *itself*. Other formulations of the optimisation problem try to address the issue with performance variability. In order to properly ensure that the designed system will perform within some specified bounds, we can instead choose a system with the best *worst-case-scenario* performance. But such a choice is usually overly conservative so several optimisation methodologies based on the quantile function of the distribution of system performance were developed. Among them, we will focus on the risk measure approach [ITIP, Chapter 12], which, again, would allow us to reduce the problem of performance-based-design into an optimisation of a real function in the case of precisely specified stochastic models. There are multiple ways how to include also the reliability in such problem specification.

When an imprecision is present in the mathematical models or when the reliability cannot be naturally included in the performance index, the problem turns into a multi-objective optimisation with objective functions being the lower and upper performance and reliability. In our talk, we will demonstrate the main features of such problem formulations, discuss the interpretation of the results, and numerical algorithms needed to obtain a solution.

References

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