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Reliability of reliability assessments

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ABSTRACT

Reliability of a system denotes the chance that it will complete its mission successfully. System is a collection of interacting components and the evaluation of its reliability depends on the knowledge of laws governing the evolution of states of the components -their failure laws-, and a relation describing how particular combinations of system states influence the state of the system -the structure function, in case when this relation is deterministic. In the classical reliability theory, only two possible states are considered for both components and the system, them being functional or failed. System reliability is then the chance that the system is functional over the whole course of its mission. Once the component failure laws are known, assessing the system reliability is simply a technical matter, but they are often not and have to be inferred by statistical methods. Our interest in this contribution lies upon investigating how we can conduct such inference on each component separately and how to integrate them into the assessment of system reliability whilst ensuring that the resulting assessment will posses proper frequency properties in the sense of type I and II errors.

Despite the success of Bayesian inference, whose resulting posterior probabilities can be straightforwardly propagated into distribution for any derived assessment, they assure proper frequency properties only asymptotically and are sensitive to the choice of prior measure in cases when only little amount of observations is available. To overcome this disadvantage, we will employ inferential methods derived from Fisher's fiducial inference. Lately, the notion of *valid* inference has been introduced in this branch of statistics to ensure proper frequency properties even for small number of observations. *Validity* represents that the inference will *unlikely* lead to incorrect results. Exactly stated, for a parameter inference in a parametric model $X \sim f_\theta$, where X represents the random observation and θ model parameters, an inference is *valid* iff

$$P_X(Bel_X(\theta \in A) \geq 1-\alpha) \leq \alpha$$

for each $\theta \notin A$, and

$$P_X(Bel_X(\theta \in A) \leq \alpha) \leq \alpha$$

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for each $\theta \in A$.
Bel_x represents a set function describing the *belief* in statements about θ given an observation x. It is the result of the conducted inference. High values of $Bel_x(\theta \in A)$ means that observed x support statement $\theta \in A$. We use the term *belief* to emphasise that the *Bel* function need not to be an additive measure as is the case in the Bayesian inference where *Bel* would be exactly the posterior distribution. Validity criterion then states that the chance that we will draw an observation x, the $P_X(.)$, which would overestimate the *belief* in incorrect statements or underestimate the same for the correct ones, is properly bounded.
The theory underlying valid inferences was described in [Martin et al., 2010], [Martin and Liu, 2013], and, more extensively, in the book [Martin and Liu, 2015]. Nevertheless, the emphasis is usually put on assuring the validity property on assertions about the model parameter. In our contribution, we will demonstrate how the same property can be achieved also for assertions about derived quantities, specifically for one sided assertions about the system reliability. We will show what are the general conditions sufficient for *valid* inference and how they manifest in specific scenarios.

References

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