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A Surrogate-Assisted Multiple Shooting Approach for Optimal Control Problems under Severe Uncertainty

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ABSTRACT

This paper presents an approach to the solution of optimal control problems under epistemic uncertainty. Traditional approaches for deterministic optimal control problems compute a nominal control policy $\mathbf{u}(t)$ to steer the state $\mathbf{x}(t)$ of a dynamical system to satisfy imposed constraints while optimising a performance index. However, in real-life applications, perfect compliance to the nominal trajectory is very hard, if possible, to achieve as uncertainty always affects any real system. Examples of this uncertainty can be imperfect state knowledge, dynamical parameters \mathbf{d} known up to a certain degree, or missed realisation of the control. To improve the solution robustness, usually design margins are considered to account for possible deviations from the reference scenario. On the other hand, an optimal control problem under uncertainty deals with this uncertainty directly from its problem statement, such that the computed control policy would have enhanced robustness and reliability from the very first design iteration.

In most real applications, the specification of a single probability measure is non-trivial, and usually a specific choice is the result of simplifying assumptions. Hence, to enlarge the model validity, in this paper we generalise a previous work on optimal control problem under precise epistemic uncertainty [Greco *et al.*, 2018] to handle imprecise random variables as described by the theory of *imprecise probability* [Augustin *et al.*, 2014]. Specifically, now the random variables $\xi \in \Omega_\xi$ have a probability distribution within a parametrised set $P(\xi) \in \mathcal{P}(\xi)$, on which no priority rule is imposed. Although replacing a single probability measure with a set results in increased level of uncertainty, we consider this case of severe uncertainty to be a more realistic representation of reality in a large number of practical applications. In this work, the uncertain variables considered are the state initial conditions $\mathbf{X}_0 \sim P(\mathbf{X}_0) \in \mathcal{P}(\mathbf{X}_0)$ and the model parameters $\mathbf{D} \sim P(\mathbf{D}) \in \mathcal{P}(\mathbf{D})$.

From here, the optimal control under imprecision addressed in this work is formulated as

$$\min_{\mathbf{u}(t) \in U} \bar{\mathbb{E}}[\phi_J] \quad (1)$$

$$\text{s.t. } \dot{\mathbf{x}} = f(t, \mathbf{x}, \mathbf{u}, \mathbf{d}) \quad (2)$$

$$\underline{\mathbb{E}}[\phi_{\mathbf{g}}(t, \mathbf{x}, \mathbf{u}, \mathbf{d})] \in \Phi_{\mathbf{g}}, \quad \bar{\mathbb{E}}[\phi_{\mathbf{g}}(t, \mathbf{x}, \mathbf{u}, \mathbf{d})] \in \Phi_{\mathbf{g}} \quad (3)$$

where \mathbf{x} is an unknown outcome of the random variable $\mathbf{X}(t)$, resulting from \mathbf{X}_0 and \mathbf{D}_0 and induced by equation (2), which describes how a point-wise trajectory realisation evolves in time. Being the state and parameter uncertain, they induce the auxiliary objective ϕ_J and constraint function $\phi_{\mathbf{g}}$ to be random variables themselves. Since the probability measure is set-valued by definition, lower- and upper-expectations are defined as

$$\underline{\mathbb{E}}[\phi] = \inf_{P \in \mathcal{P}} \mathbb{E}_P[\phi] \quad \bar{\mathbb{E}}[\phi] = \sup_{P \in \mathcal{P}} \mathbb{E}_P[\phi]. \quad (4)$$

Therefore, the optimisation is formulated to minimise the upper bound (worst-case) of the expectation operator on the objective (1), while the constraints (3) impose all the range of expected values to lay within the convex set $\Phi_{\mathbf{g}}$. It is worth underlining how the expectation formulation encloses common cases of constraints (or objective) in *expected value*, e.g. $\phi_{\psi}(t_f, \mathbf{x}_f) = \mathbf{x}_f$, or in *probability*, e.g. $\phi_{\psi}(t_f, \mathbf{x}_f) = \mathbb{I}_A(\mathbf{x}_f)$.

In general, the optimal control problem under imprecision is infinite-dimensional with no closed-form solution. Hence, the dynamical optimal control problem is converted into a static constrained optimisation, which is then possible to solve with black-box numerical routines, e.g. local NLP solvers. This finite dimensional conversion is realised by means of a transcription scheme. In this paper, we present a novel transcription for optimal control problems under uncertainty that extends the classical direct multiple shooting transcription to account for random variables defined on extended sets. The proposed approach employs a Generalised Intrusive Polynomial Expansion to model and propagate uncertainty within each sub-segment of the multiple shooting, resulting in a chain of polynomial surrogates $\tilde{F}_{\bar{t}} : \Omega_{\xi} \rightarrow \mathbb{R}^{n_s}$ that maps the uncertain initial conditions and parameters to the state vector at a future time \bar{t} .

This inexpensive surrogate is then used to reduce the computational complexity in the computation of the expectations in (1)-(3), which require a further inner loop of optimisation on the set of probability measures to find the requested bounds. In particular, for the generic density function $p(\xi)$, the expectation in (4) is approximated with a quadrature scheme as

$$\mathbb{E}_p[\phi] = \int_{\Omega_{\xi}} \phi(\xi) p(\xi) d\xi \approx \sum_{j=1}^N w_j \phi(\xi_j) p(\xi_j). \quad (5)$$

This approach requires the computation of $\phi(\xi_j)$ only once for an inner optimisation, reducing considerably its computational burden to the cheap evaluation of the different densities $p(\xi)$ within the imprecise set $\mathcal{P}(\xi)$.

In this paper, the developed approach is then applied to the design of a full 3D robust low-thrust trajectory with uncertain initial conditions.

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