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AK-MCS extension for the Efficient Estimation of Extreme Quantiles and Failure Probabilities

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ABSTRACT

We consider the problem of estimating a probability of failure pf, defined as the volume of the excursion set of a complex (e.g. output of an expensive-to-run finite element model) scalar performance function J below a given threshold, under a probability measure that can be recast as a multivariate standard gaussian law using an isoprobabilistic transformation. We focus on a method able to deal with problems characterized by multiple failure regions, possibly very small failure probability pf (say $\sim 10^{-6}$ - 10^{-9}), and when the number of evaluations of J is limited. The present work is an extension of the popular Kriging-based active learning algorithm known as AK-MCS, as presented in [Schobi et al.]. Similarly, the problem is formulated in such a way that the computation of both very small probabilities of failure and extreme quantiles is unified. The key idea merely consists in replacing the Monte-Carlo sampling, used in the original formulation to propose candidates and evaluate the failure probability, by a centered isotropic Gaussian sampling in the standard space, which standard deviation is iteratively tuned. This extended AK-MCS (eAK-MCS) method inherits its former multi-point enrichment algorithm allowing to add several points in each iteration, and, due to the Gaussian nature of the surrogate, to estimate a failure probability/quantile range at each iteration step. An Importance Sampling procedure allows to accurately estimate a quantile of level as low as 10^{-9}, with a reasonable number of calls to the surrogate ($\sim 10^7$). A batch strategy on the quantile permits to further exploit the parallelization capability of eAK-MCS. Similarly to other Kriging-based methods, the proposed approach suffers from the same limitations, namely the low input dimension and depends strongly on the ability of the surrogate to fit the performance function. We illustrate the performances of the proposed method on several two, six and eight-dimensional benchmark analytical functions, for which the failure probability is very low (pf $\sim 10^{-6}$ - 10^{-9}), including in particular the three test-cases presented in [Bect et al.] (where a Bayesian Subset Simulation approach is proposed). In the context of pure failure probability estimation, there is no clear advantage of eAK-MCS over BSS [Bect et al.]. Its main asset lies in the fact that most AK-MCS based algorithms could be possibly adapted in the context of extreme event, which is illustrated with the proposed extreme quantile estimation tool. In the test-cases studied, small failure probabilities and quantiles are

estimated with a satisfactory accuracy with a moderate number evaluations of the performance function J ($\sim$ 25-140), with a batch strategy.

References

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