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Level Set Methods and Frames for Nonlinear Stochastic Problems

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ABSTRACT

The solutions to many engineering problems described by conservation laws exhibit non-smooth dependence on material parameters. In many applications, e.g., large scale CO₂ storage in aquifers [Nordbotten *et al.*, 2012], the material parameters are unknown due to lack of data and inability to accurately resolve small scale variations. The combination of discontinuous dependence on data and uncertainty poses challenging problems when it comes to efficiently estimating quantities of interest. Efficient stochastic representation (e.g., spectral expansions) of quantities of interest requires knowledge of the location of the discontinuities in stochastic space. In this work, we introduce a framework for tracking discontinuities in stochastic space using a level set formulation, followed by reconstruction of quantities of interest with localized basis functions.

Level set methods can be used to track deforming interfaces and are attractive due to the flexibility with respect to the geometry of the regions separated by the interfaces. For instance, they are not restricted to star-convex regions and are therefore of interest in solving complex discontinuous problems with moderate stochastic dimensionality. In this work, we use a level set formulation to identify the location of solution discontinuities. A set of realizations of the nonlinear conservation law of interest is treated as an image, and discontinuities are tracked in pseudo-time by the classical image segmentation techniques introduced in [Malladi *et al.*, 1994].

The zero level set of the steady-state level set function is made to coincide with the discontinuities in stochastic parameter space of the function of interest, as shown in Figure 1. The function of interest (denoted u) is piecewise smooth in regions where the level set function is strictly positive, and strictly negative, respectively.

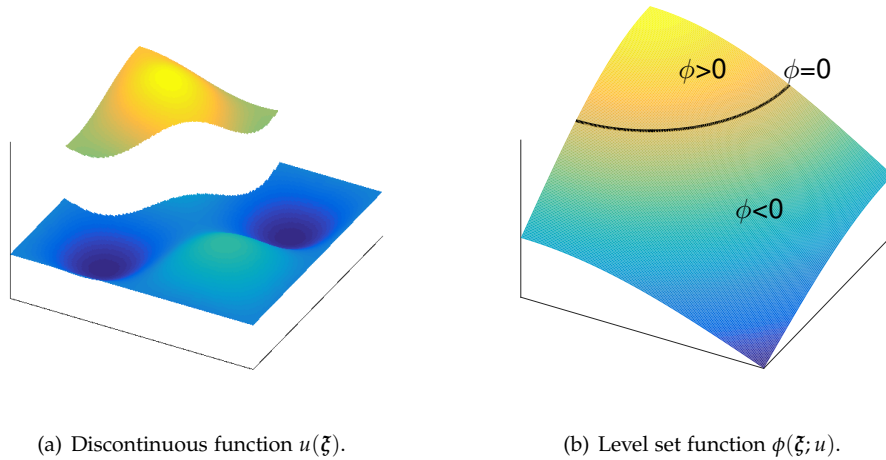


Figure 1: Discontinuous function $u(\xi)$ (for fixed space and time) and an associated level set function ϕ with the zero level set being equal to the location of the discontinuity in u . The uncertainty is parameterized by the random vector ξ .

The computed level set solution yields an approximation of the location of conservation law discontinuities. One can see this as a classification problem, where some conservation law realizations may be misclassified due to inexact level set solutions. Because of the irregular shape of the piecewise continuous solution regions, a surrogate for the true solution is constructed using frames, i.e., in this case polynomial functions restricted to each of the different solution regions. Robust computation of the frame coefficients is achieved using the Least Absolute Deviations method [Charnes *et al.*, 1955] to compensate for misclassification of solution realizations, and compared to standard Least Squares Methods.

The performance of the methodology is demonstrated on nonlinear problems from computational fluid dynamics and CO₂ storage, and compared to existing adaptive multi-element generalized polynomial chaos methods [Wan and Karniadakis, 2005]. The proposed method yields a significant speedup in terms of the number of calls to an expensive conservation law solver, at the added cost of solving a level set equation in random space and pseudo-time.

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