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Surrogate-based inversion for first-arrival seismic tomography

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ABSTRACT

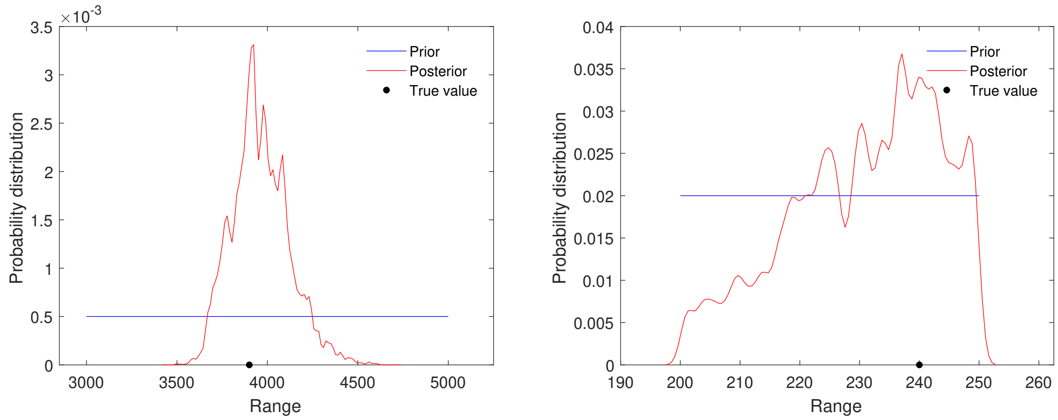
Seismic tomography is used to infer the subsurface seismic structure of a medium from observations of seismic waves. We focus on active tomography where a network of sources generates seismic waves propagating in the subsurface and recorded at seismic stations. The structure of the medium is described by a vector of parameters, denoted $\mathbf{m}$, containing the wave velocities and geometrical informations to be inferred from the observations. Specifically, we consider a Bayesian inversion where a set of first-arrival times, collected in the vector $\mathbf{t}_{\text{obs}}$, is used to infer the parameters using the the Bayes' theorem [Bayes, 1763],

$$p(\mathbf{m}|\mathbf{t}_{\text{obs}}) \propto p(\mathbf{t}_{\text{obs}}|\mathbf{m})p(\mathbf{m}). \quad (1)$$

Here, the vertical bar $\cdot|\cdot$ means "conditional on", and we have denoted $p(\mathbf{m})$ the prior distribution of the parameters, $p(\mathbf{t}_{\text{obs}}|\mathbf{m})$ the likelihood of the arrival times and $p(\mathbf{m}|\mathbf{t}_{\text{obs}})$ the posterior distribution of the parameters.

The likelihood requires a comparison between the measured arrival times and their values predicted by the model for given parameter values in $\mathbf{m}$. An Eikonal solver [Noble *et al*, 2014] can be used to predict the arrival times; however, the computational cost of the Eikonal solvers is too large to permit the extensive sampling of the posterior of $\mathbf{m}$, for instance using a Markov Chain Monte Carlo (MCMC) algorithm [Metropolis *et al*, 1953, Hastings, 1970]. To circumvent this issue, we rely on polynomial chaos expansions [Le Maître and Knio, 2010] as a surrogate of the forward Eikonal solver in the inversion algorithm, therefore saving the computational burden of its computation.

We present results of initial tests using synthetic data. These tests show a satisfactory inference of the parameters, as illustrated in Fig. 1. Plotted are the prior and posterior distributions of two velocity model parameters, namely i) the wave velocity of a geological layer and ii) the location of a geological interface between two layers. The true parameter values used to generate the synthetic data are also reported. We observe that the maximum a posteriori (MAP) values are close to these true values. The narrow velocity posterior means that the observations are very informative for this parameter, when the flatter interface location posterior denotes that it remains more uncertain.



(a) *First layer wave velocity* **(b)** *First interface location*

Figure 1: *Example of Bayesian calibration.*

Our numerical tests also reveal that the location of the sources and recording stations can affect the number of Eikonal model evaluations necessary to build accurate surrogates of the arrival times, in particular by inducing non-smooth dependencies of the arrival times with $\mathbf{m}$. Another challenging question concerns the design of the sources and stations network in order to enhance the parameter calibration.

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