

# Different measure approximations for efficient constrained multi-objective optimization under uncertainty

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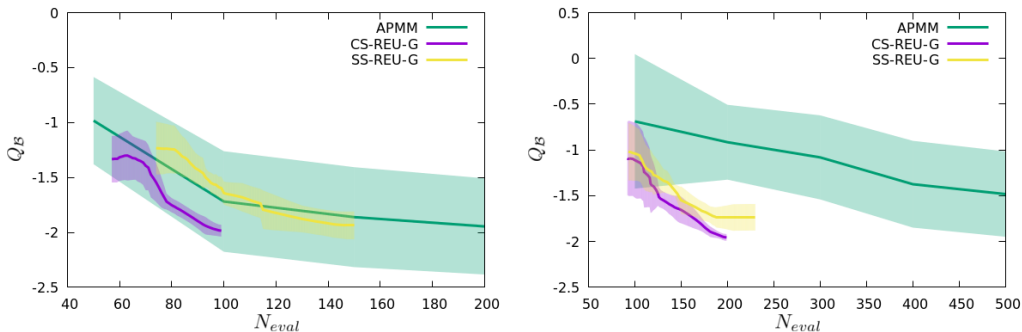
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**Keywords:** Uncertainty quantification, optimization, robustness, reliability.

## ABSTRACT

The SABBa framework has been shown to tackle multi-objective optimization under uncertainty problems efficiently. It deals with robust and reliability-based optimization problems with approximated robustness and reliability measures. The recursive aspect of the Bounding-Box (BB) approach has notably been exploited in [1], [2] and [3] with an increasing number of additional features, allowing for little computational costs. In these contributions, robustness and reliability measures are approximated by a Bounding-Box (or conservative box), which is roughly a uniform-density representation of the unknown objectives and constraints. It is supplemented with a surrogate-assisting strategy, which is very effective to reduce the overall computational cost, notably during the last iterations of the optimization.

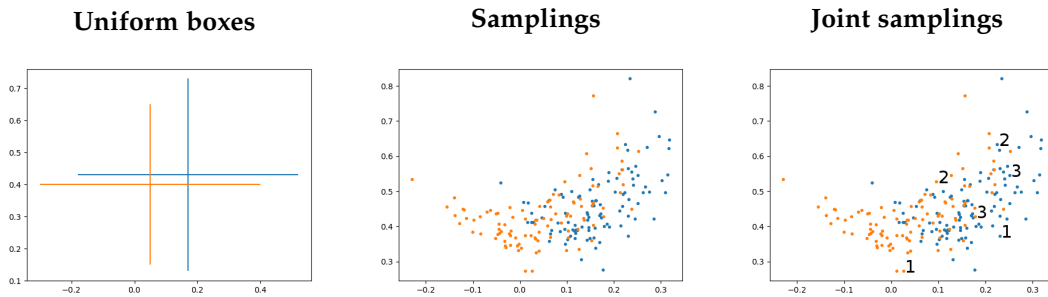
In [3], SABBa has been quantitatively compared to more classical approaches with much success both concerning convergence rate and convergence robustness.



**Figure 1:** Cost comparison between APMM and SABBa with a) high-quality or b) low-quality surrogate model.

Figure 1 shows better mean performance and significantly reduced variability of the SABBa CS-REU-G method w.r.t an A Priori MetaModel strategy. SABBa allows for local refinement, hence higher robustness to low-quality surrogate model.

We propose in this work to further improve the parsimony of the approach with a more general framework, SAMMA (Surrogate-Assisted Multi-fidelity Measure Approximation), allowing for objects other than Bounding-Boxes to be compared in the recursive strategy. Such non-uniform approximations have been proposed in previous works like [4] and [5]. Among others, sampling and Gaussian measure approximations are presented and quantitatively compared in the following. We propose suitable Pareto dominance rules and POP (Pareto Optimal Probability) computations for these new measure approximations. Potential gains in terms of discriminance between boxes of the sampling measure approximation and joint information are depicted in Table 1.



Pareto Optimal Probability			
Design	Uniform	Sampling	Joint
Blue	0.7388	0.4446	0.12
Orange	0.8455	0.868	1

**Table 1:** Potential discriminative gains

To extend the framework applicability to complex industrial cases, and alongside the multi-fidelity between different UQs (Uncertainty Quantification) inherent to the recursive strategy, we propose to plug multi-fidelity approaches within the measure computations. This approach should allow tackling very complex industrial problems in an acceptable timeframe.

## REFERENCES

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